Joy, Ch 1, "From Fish to Infinity"

Small Group Discussion

① What did the author say?

- Notes on board: key phrases, sentences

② What do you think he meant?

Class Discussion

② What do you think he meant by ...?

③ What do you make of this? Surprizes? Reactions ...

Joy, Ch 2, "Rock Groups"

Each number is represented by a group of "rocks"!

① Squares

② Evens & Odds

③ Odd + Odd = Even

④ Rectangles

⑤ Sum of consecutive odds

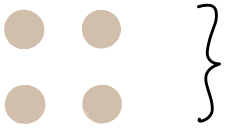
⑥ Sum of consecutive numbers

Squares

Which groups of rocks can be arranged in a square pattern?

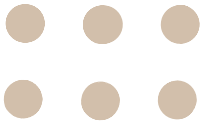
- tidy rows & columns
- same number of rows & columns

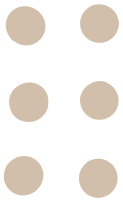
· Group of 4 :



} 2 rows, 2 columns : _____

· Group of 6 :



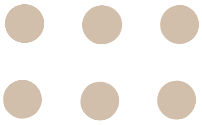


_____ rectangle _____ rectangle ...

A group of 6 rocks _____ be arranged in a square pattern.

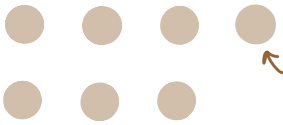
Evens & Odds

A group of rocks that can be arranged in _____ has an ____ number of rocks.



6 is an ____ number.

A group of rocks that _____ be so arranged has an ____ number of rocks.

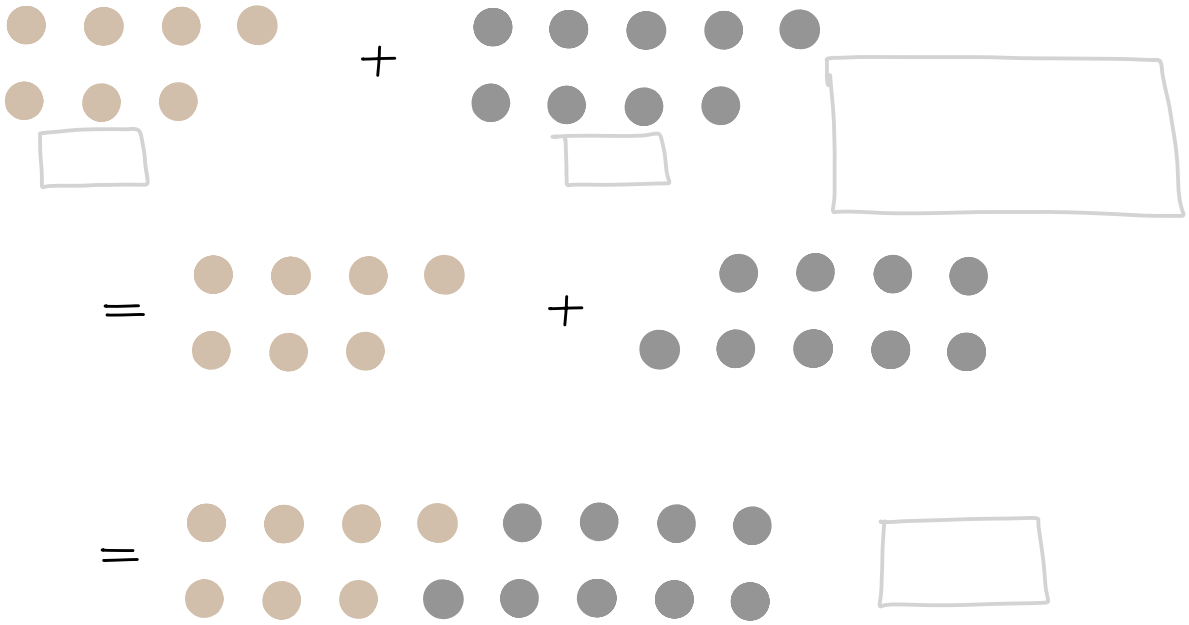


protuberance

7 is an ____ number

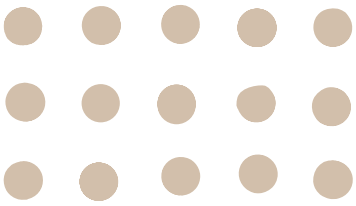
Odd + Odd = Even

Why? Arrange the rock groups so that the protuberances match up.



Rectangles

Allow more than 2 rows: some odd numbers ____ be arranged in a _____ pattern:



15 rocks in a _____ rectangle

- For a given number of rocks, _____ are there to arrange the rocks in a rectangular pattern?
- What does the number of _____ tell us about the number of _____?

  ↳ Activity for next class.

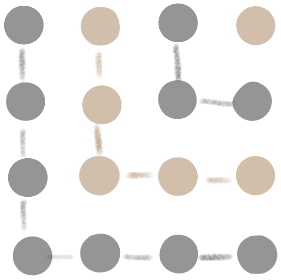
Sum of Consecutive Odds

$1 + 3 + 5 + 7 = ?$

$1 + 3 + 5 + \ldots + 19 = ?$

$1 + 3 + 5 + \ldots + 999 = ?$

Key Idea: Build a _____.

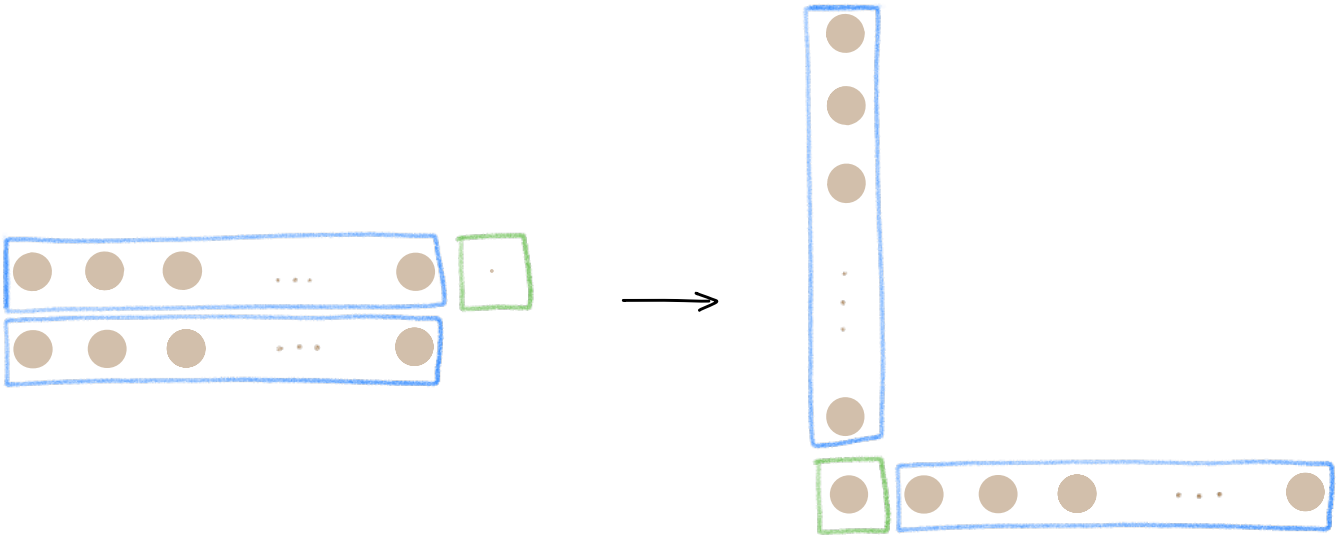


$1 + 3 + 5 + 7 = \square = \square$

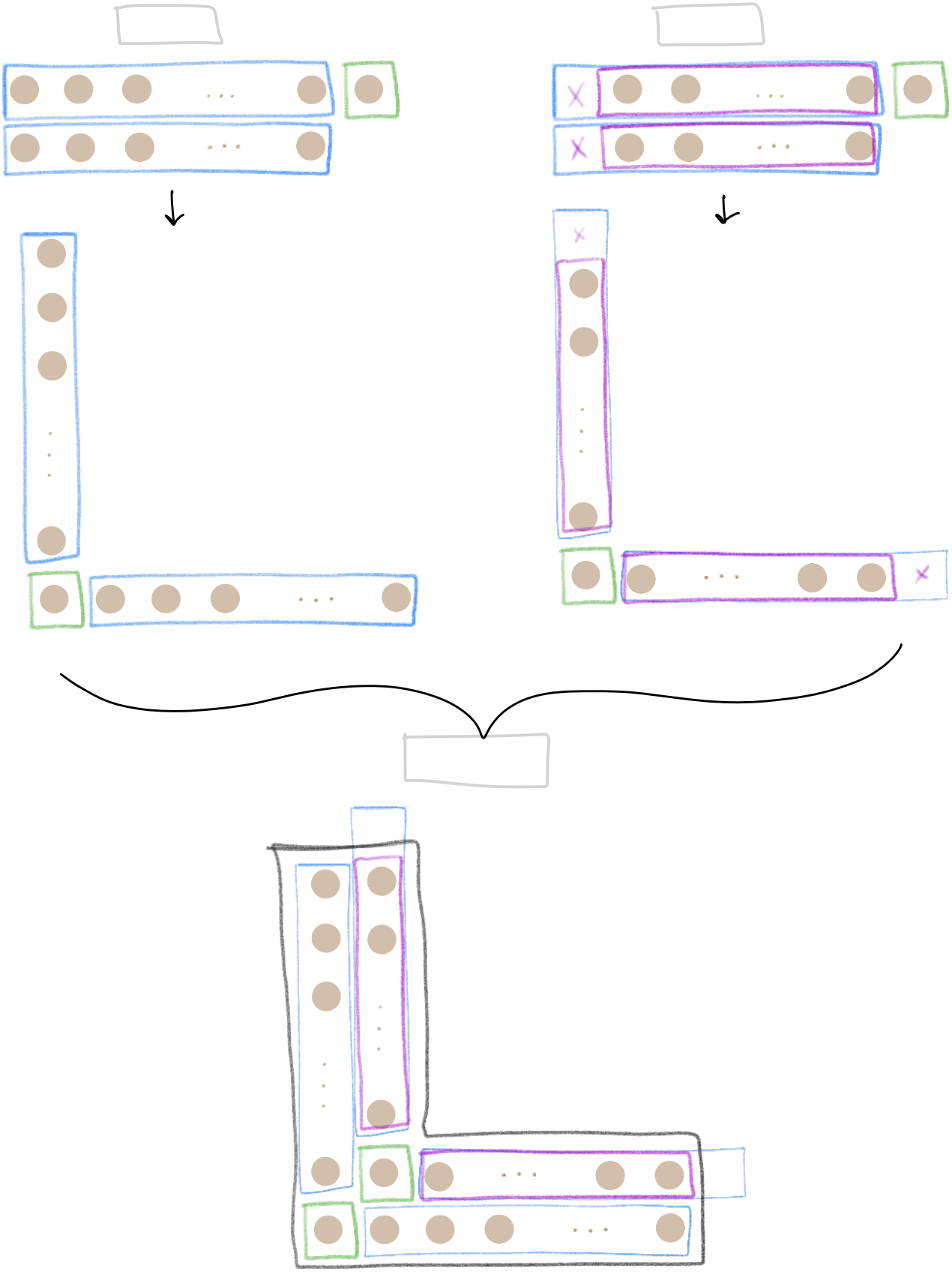
Conjecture: The sum of consecutive odd numbers is always a _____ number.

Key Idea: Any ____ number, no matter how large, can be arranged into a "_____" shape: an ell shape (L) with the ____ equal to the _____. Why?

- Remove the protuberance so you have two ____ rows.
- Rearrange one of them _____, on the ____. Then put the protuberance in the _____:



Since consecutive odd numbers _____, they fit together perfectly to form thicker "square ell."

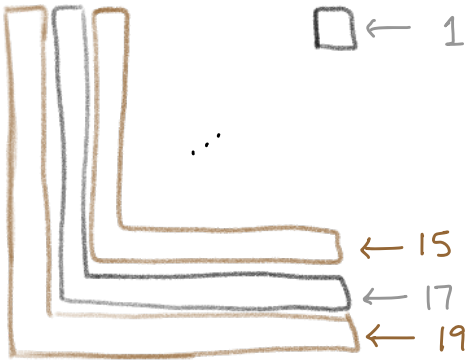


This _____ the sum of consecutive odd numbers is always a square number. Now our _____ has become a _____.

Now that we understand why our conjecture is true, we can use this understanding to calculate longer sums.

Example: $1 + 3 + \ldots + 17 + 19 = ?$

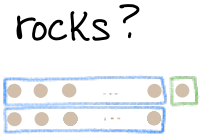
Start with 19 rocks. Make "square ell". This will be the ell on the _____. Build up the square with _____ "square ells": 17 rocks, 15 rocks, and so on.

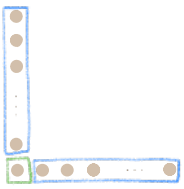


How many rocks total?

Need to figure out how many rocks _____.

How big is the "square ell" with 19 rocks?





- Separate protuberance: _____
- Divide into two rows: _____
- Form "square ell" with base & height of: _____

Therefore the outside ell has _____ in its base, and the whole square has _____ rocks.

We conclude that:

$$1 + 3 + 5 + \ldots + 19 = \_\_\_\_$$

Challenge: Use this trick to calculate a larger sum:

$$1 + 3 + 5 + \ldots + 999 = ?$$